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ADSORPTION ANALYSIS: EQUILIBRIA AND KINETICS

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Dedication

I dedicate this book to my parents.

Preface

The significant research in adsorption in the 70s through the 90s could be attributed to the discovery of many new porous materials, such as carbon molecular sieve, and the invention of many new clever processes, notably Pressure Swing Adsorption (PSA) processes. This evolution in adsorption research is reflected in many books on adsorption, such as the ones by Ruthven (1984), Yang (1987, 1997), Jaroniec and Madey (1988), Suzuki (1990), Karger and Ruthven (1992) and Rudzinski and Everett (1992). Conferences on adsorption are organized more often than before, such as the Fundamentals of Adsorption, the conference on Characterization of Porous Solids, the Gas Separation Technology symposium, the Symposium in Surface Heterogeneity, and the Pacific Rim workshop in Adsorption Science and Technology. The common denominator of these books and proceedings is the research on porous media since it is the heart for the understanding of diffusion and adsorption. Since porous media are very complex, the understanding of many practical solids is still far from complete, except solids exhibiting well defined structure such as synthetic zeolites. It is the complex interplay between the solid structure, diffusion and adsorption that makes the analysis of adsorption more complicated than any other traditional unit operations process such as distillation, etc.

Engineers dealing with adsorption processes, therefore, need to deal with model equations usually in the form of partial differential equation, because adsorption processes are inherently transient. To account for the details of the system, phenomena such as film diffusion, interparticle diffusion, intragrain diffusion, surface barrier and adsorption in addition to the complexities of solid structure must be allowed for. The books of Ruthven, Yang, and Suzuki provide excellent sources for engineers to fulfill this task. However, missing in these books are many recent results in studying heterogeneous solids, the mathematics in dealing with differential equations, the wider tabulation of adsorption solutions, and the many methods of

measuring diffusivity. This present book will attempt to fill this gap. It starts with five chapters covering adsorption equilibria, from fundamental to practical approaches. Multicomponent equilibria of homogeneous as well as heterogeneous solids are also dealt with, since they are the cornerstone in designing separation systems.

After the few chapters on equilibria, we deal with kinetics of the various mass transport processes inside a porous particle. Conventional approaches as well as the new approach using Maxwell-Stefan equations are presented. Then the analysis of adsorption in a single particle is considered with emphasis on the role of solid structure. Next we cover the various methods to measure diffusivity, such as the Differential Adsorption Bed (DAB), the time lag, the diffusion cell, chromatography, and the batch adsorber methods.

It is our hope that this book will be used as a teaching book as well as a book for engineers who wish to carry out research in the adsorption area. To fulfill this niche, we have provided with the book many programming codes written in MatLab language so that readers can use them directly to understand the behaviour of single and multicomponent adsorption systems

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January 1998

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